Meeting 1 Math 22

Todays agenda

1. "Setting the Stage" Reflection Activity
2. Tour of the course:
   - Syllabus
   - Webpage
   - Canvas
   - Gradescope
   - Edfinity
   - Active Calculus Textbook
3. Start learning calculus!

Preview Activity 9.1.1

- Read section 9.1 up to Activity 9.1.1.
- Complete 9.1.1 and discuss w/ your group.
- Class discussion.

(a) Evaluate $A(1000,7) = 1000e^{.05 \cdot 7}$ using a calculator.

(b) $A(5000,8) = 7{,}459.12$ How much money I will have after investing \$5000 for 8 years.

(c) If we fix $P=1000$, we get a single variable function $f(t) = 1000e^{.05t}$. Fill table using a calc.

(d) If we fix $t=10$, we get a single variable function $g(P) = Pe^{.05 \cdot 10}$. Fill table using a calc.

(e) We are asked to solve the eq.

$$10{,}000 = A(P,t) = Pe^{0.05t}$$

for P and t. One could find specific solutions by fixing P or t and solving for the other.

Section 9.1.1 Functions of Several Variables

Definition 9.1.3 A function f of two independent variables is a rule that assigns to each pair (x,y) of real numbers in some set D a unique real number $f(x,y)$. ∎

Examples:
- $A(P,t)$
- $f(x,y) = x^2+y^2+\sin(xy)$.

Definition 9.1.4 The domain of a function f is the set of all inputs for which the function is defined.

Example: Domain of $A(P,t) = Pe^{.05t}$
$= \{(P,t) \mid P \geq 0, t \geq 0\}$
since P is a dollar amount and t is time.

Domain of $f(x,y)$ is $\mathbb{R}^2$ = all ordered pairs of real numbers since the expression $x^2+y^2+\sin(xy)$ makes sense for all values of x and y.

Activity 9.1.2

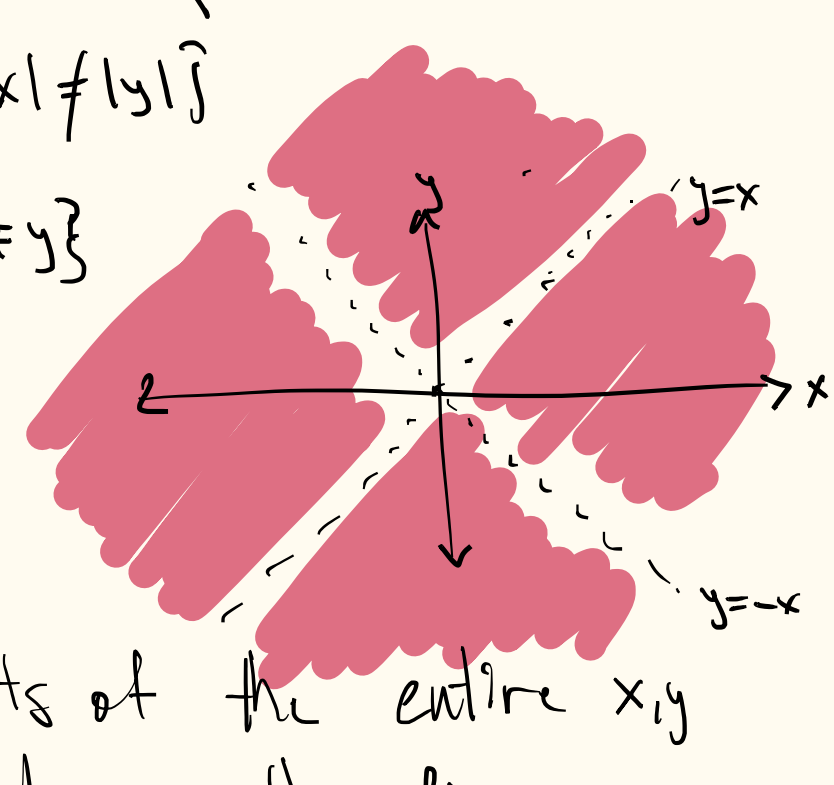
- Complete 9.1.2 and discuss w/ your group.
- Class discussion.

a. $f(x,y) = x^2+y^2$
$\text{Dom}(f) = \mathbb{R}^2$

b. $f(x,y) = \sqrt{x^2+y^2}$
$\text{Dom}(f) = \mathbb{R}^2$ Since $x^2+y^2 \geq 0$.

c. $Q(x,y) = \dfrac{x+y}{x^2-y^2}$ The only restriction is that we cant divide by 0.

$\text{Dom}(Q) = \{(x,y) \mid x^2 \neq y^2\}$
$= \{(x,y) \mid |x| \neq |y|\}$
$= \{(x,y) \mid x \neq y \text{ and } -x \neq y\}$



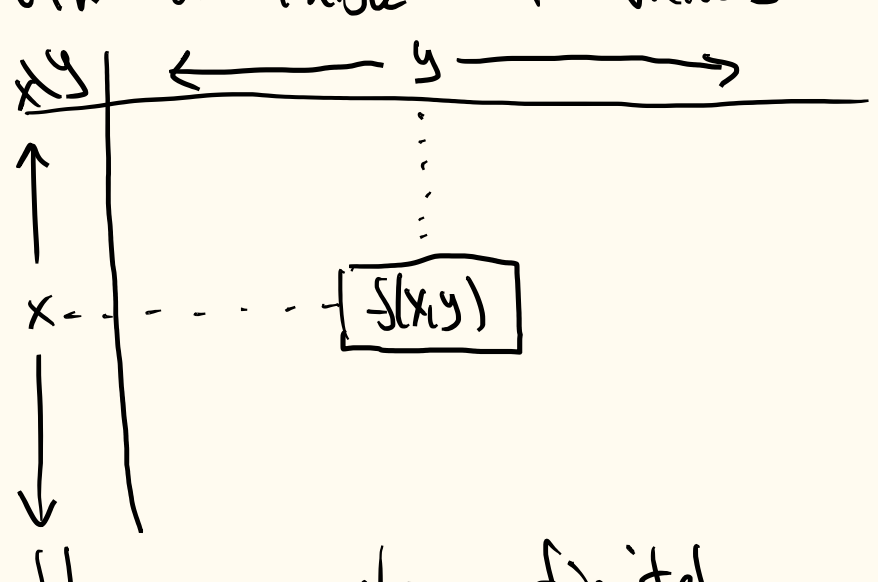
The domain consists of the entire x,y plane except points on the line y=x or y=-x

d. $s(x,y) = \dfrac{1}{\sqrt{1-xy^2}}$ We cant divide by 0 or take the $\sqrt{\ }$ of a negative number.

$\text{Dom}(s) = \{(x,y) \mid 1-xy^2 > 0\}$

Section 9.1.2 Representing Functions of Two Variables

the primitive way to represent a function is with a table of values



the table represents finitely many inputs and outputs $(x, y, f(x,y))$ of the function.

Activity 9.1.3

- Complete as a class.

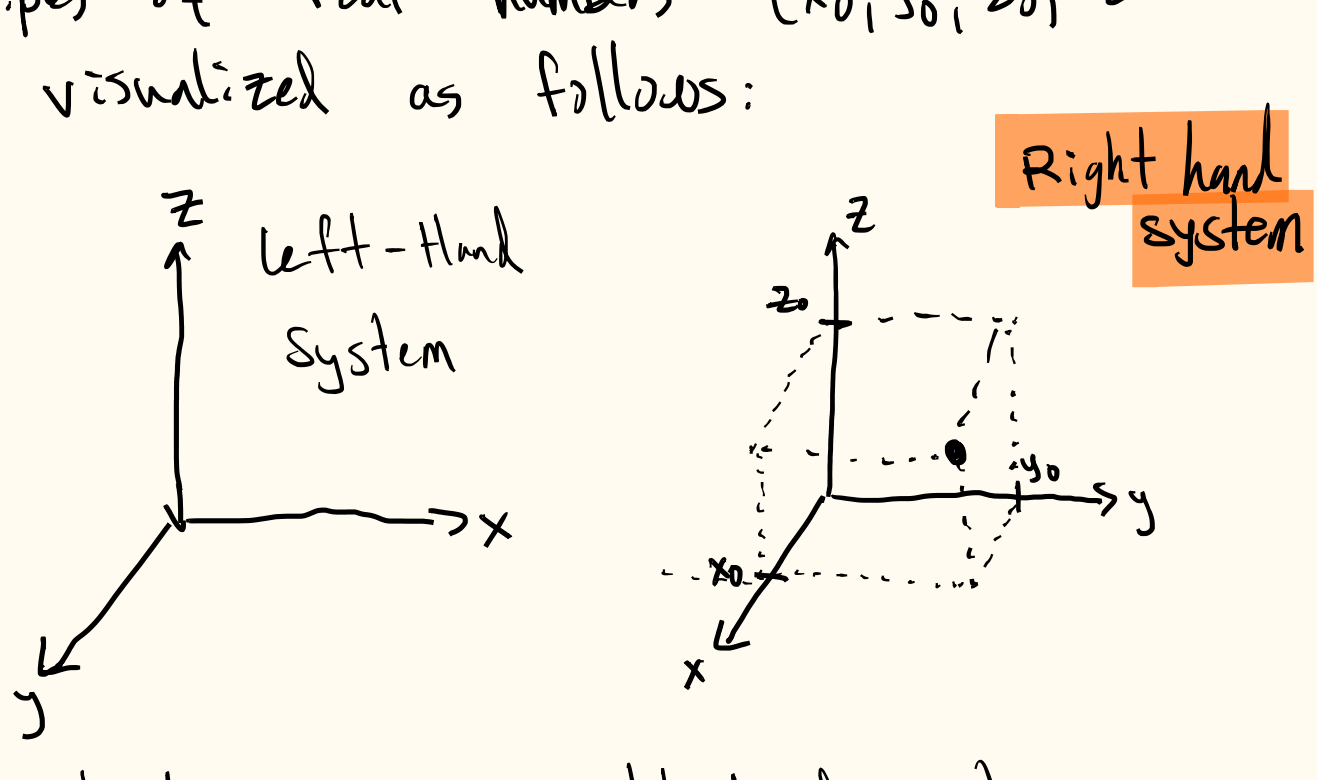
Table 9.1.5. Values of $f(x,y) = \frac{x^2\sin(2y)}{y}$

$x\backslash y$	0.2	0.4	0.6	0.8	1.0	1.2	1.4
25	7.6	14.0	18.2	19.5	17.8	13.2	6.5
50	30.4	56.0	72.8	78.1	71.0		26.2
75	68.4		163.8	175.7	159.8	118.7	58.9
100	121.7	224.2	291.3	312.4	284.2	211.1	104.7
125	190.1	350.3	455.1		444.0	329.8	163.6
150	273.8	504.4	655.3	702.8	639.3	474.9	235.5
175	372.7	686.5	892.0	956.6	870.2	646.4	
200	486.8	896.7	1165.0	1249.5	1136.6	844.3	418.7
225	616.2	1134.9	1474.5	1581.4	1438.5	1068.6	530.0
250	760.6	1401.1		1952.3	1776.0	1319.3	654.3

→ $f(125, .8)$

Activity 9.1.3. Complete Table 9.1.5 by filling in the missing values of the function f. Round entries to the nearest tenth.

Triples of real numbers (x_0, y_0, z_0) can be visualized as follows:



The book uses a right-hand system. You can remember the order of the axes using the right hand rule: index finger = x-axis, middle finger = y-axis, thumb = z-axis.

Definition 9.1.6 The graph of a function $f(x,y)$ is the set of all points $(x, y, f(x,y))$ where (x,y) is in the domain of f. ∎

A table of function values is a finite collection of points on a graph. the graph includes infinitely many points and is usually graphed w/ a computer.

Try Geogebra 3D

Section 9.1.3 Some Standard Equations in $\mathbb{R}^3$

In $\mathbb{R}^2$, equations of the form $x=c$ or $y=c$ are lines perpendicular to the coordinate axes. What do these eq. look like in $\mathbb{R}^3$?

Activity 9.1.4

- Complete Activity 9.1.4 and discuss w/ your group.
- Class discussion.

Conclusion: equations like $x=c$ are planes perpendicular to the coordinate axes. When $c=0$, we get the coordinate planes

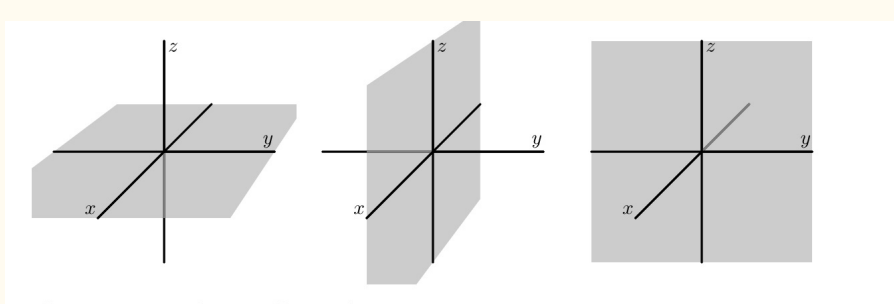
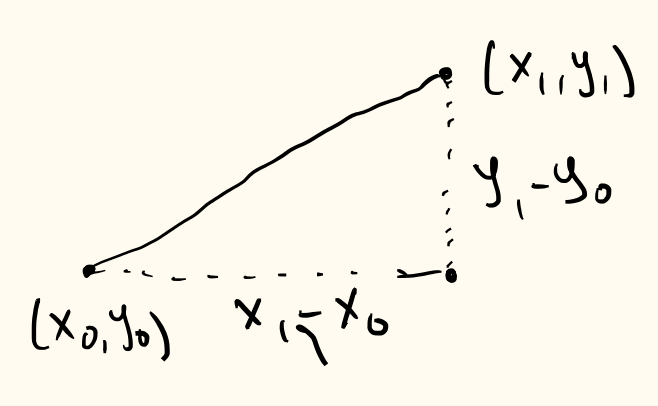


Figure 9.1.11. The coordinate planes.

Activity 9.1.5

- Complete Activity 9.1.5 and discuss w/ your group.
- Class discussion.

In two dimensions distance between (x_0, y_0) and (x_1, y_1) is



$$\sqrt{(x_1-x_0)^2 + (y_1-y_0)^2}$$

by Pythagorean Theorem.

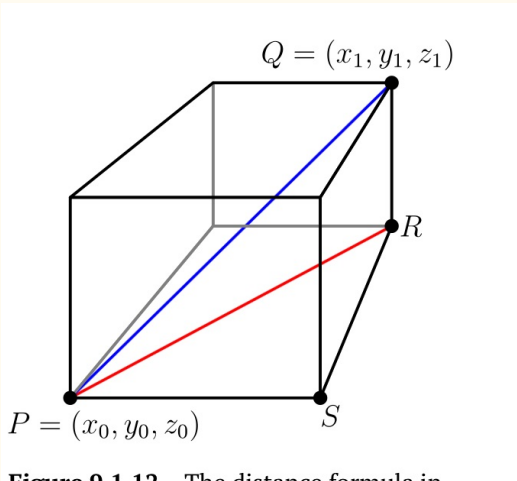
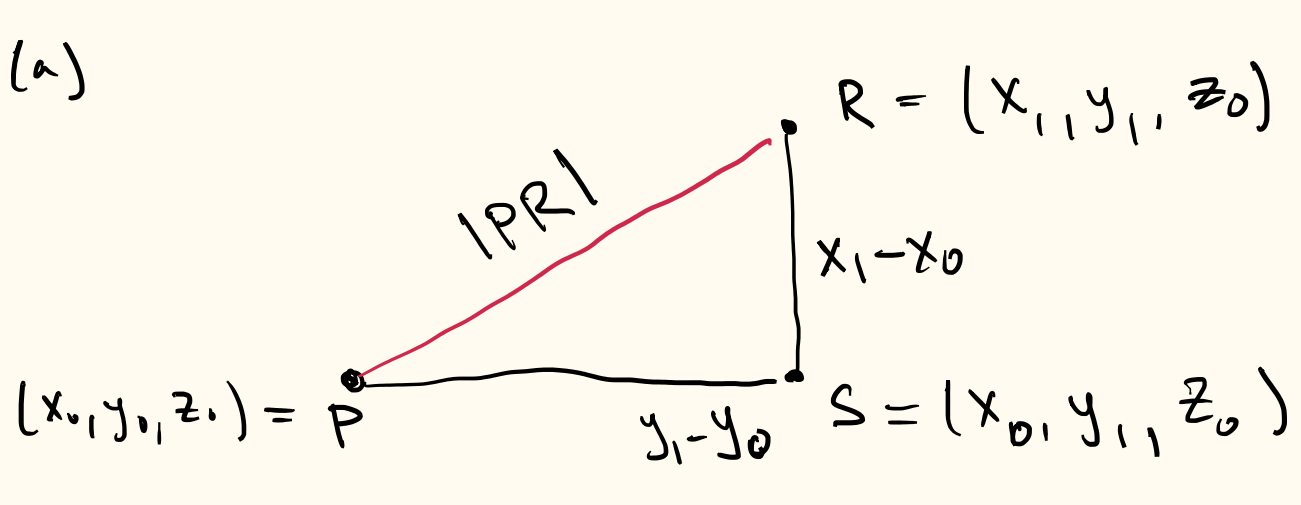


Figure 9.1.12. The distance formula in $\mathbb{R}^3$.

Using the right hand rule:
- the segment PS lies on the y-axis
- the segment SR lies on the x-axis
- the segment RQ lies on the z-axis.

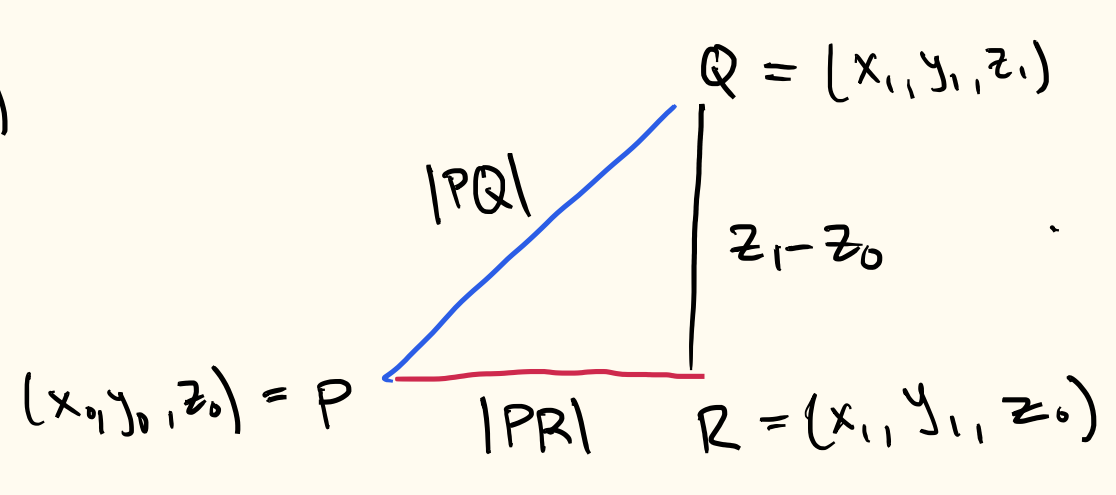
(a)



Using the Pythagorean theorem,

$$|PR| = \sqrt{(y_1-y_0)^2 + (x_1-x_0)^2}$$

(b)



$$|PQ| = \sqrt{|PR|^2 + (z_1-z_0)^2}$$

$$= \sqrt{(x_1-x_0)^2 + (y_1-y_0)^2 + (z_1-z_0)^2}$$

this is the formula for finding the distance between two points in $\mathbb{R}^3$.